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The Standard Quantum Limit for the Li-Baker HFGW Detector

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Abstract. The Standard Quantum Limit defines the lower sensitivity limit for all measurement instruments, including gravitational-wave detectors, according to the Heisenberg uncertainty principle In this paper, the standard quantum limit (SQL) is explored for the Li-Baker synchro-resonance HFGW detector design. The effects of quantum back action are scaled using an estimate of contained energy within the detector, and various sources of the Q-factor, or quality factor, are described as arising from signal integration time, antenna gain of the detector's geometric arrangement, and the signal selectivity offered through the use of fractal membranes. An overall sensitivity limit is calculated for the Li-Baker detector design, and is compared and contrasted with a similar limit for the Gertsenshtein-effect detector alone. It is concluded that the Li-Baker detector is not quantum noise limited.

Keywords: Gravitational waves, HFGW, standard quantum limit, SQL, quantum back action, Heisenberg uncertainty principle, Q-factor, quality factor, Li-Baker, Gertsenshtein, detector
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INTRODUCTION TO SQL & BACKGROUND

The Standard Quantum Limit (SQL) will be introduced and reviewed in this section, and design of the Li-Baker HFGW Detection System will also be reviewed to understand how the SQL might limit the sensitivity of this new type of GW detector.

Review of the Standard Quantum Limit

The Standard Quantum Limit (SQL) is often defined as “The limit on measurement accuracy at quantum scales due to back-action effects.” But what is “back-action”? From Clerk (2008) “Quantum Noise & Quantum Measurement,” the Heisenberg Uncertainty Principle is:

$$(\Delta x) \times (\Delta p) > \hbar/2 \tag{1}$$

where Δx is the position uncertainty, Δp is the momentum uncertainty, and $\hbar$ is Plank’s constant divided by 2π. Thus measuring x disturbs p, which in turn disturbs future measurements of x:

$$\Delta x(dt) = \Delta x(0) + dt[\Delta p(0)/m] \tag{2}$$

Where $\Delta x(0)$ is the initial position uncertainty is , $\Delta p(0)$ is the initial momentum uncertainty, dt is the time of the future measurement, and m is the mass of the system under measurement. E/c2 may be substituted for mass in an energy only system.

To summarize, the quantum effects of measurements on future measurements is quantum back action. Therefore the Standard Quantum Limit defines the lower sensitivity limit for all measurement instruments, including gravitational-wave detectors, according to the Heisenberg uncertainty principle. Detectors can not avoid quantum back action, however the use of higher energies in the detection process can change the relative scale and impact of back action, and the use of squeezed states can shift the relative distribution of back action into states not involved in measurement.

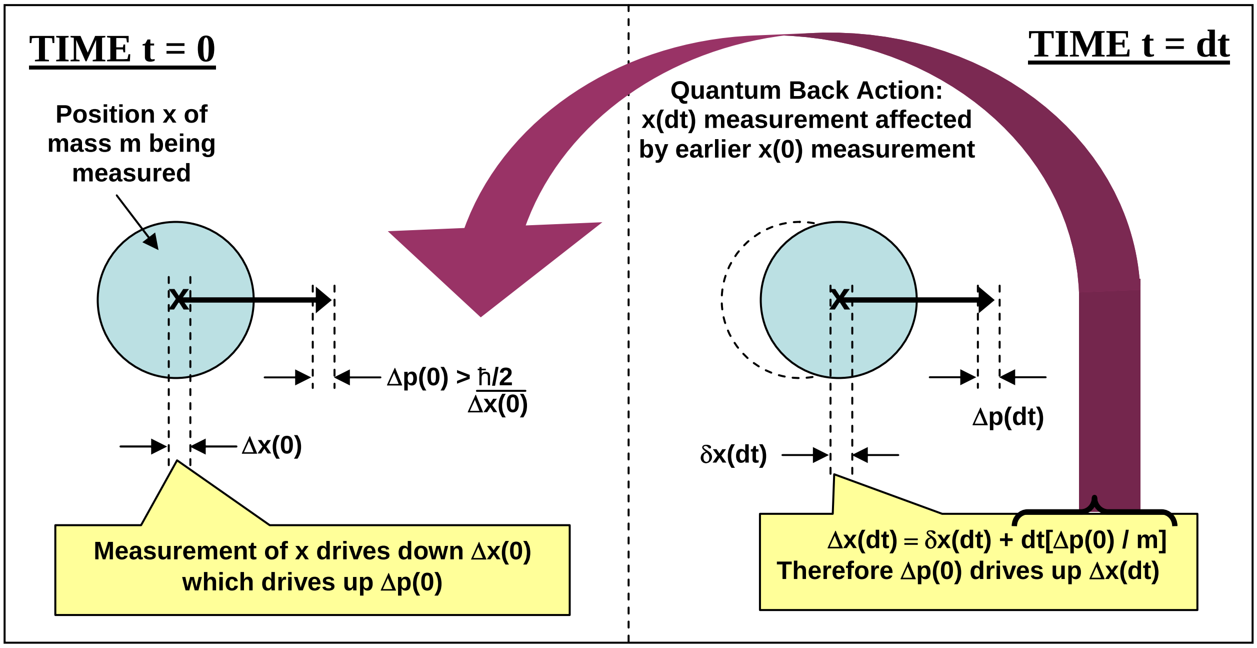

FIGURE 1. An example of how quantum back action is the mechanism for creating the standard quantum limit

Review of the Li-Baker HFGW Detector Design

The basic concept of the Li-Baker detector (Li *et al.*, 2008; Baker, Stephenson and Li, 2008) is a synchro-resonance solution of the Einstein equations, related to but not the same as the Gertsenshtein effect (or inverse Gertsenshtein effect). In his classic paper, Gertsenshtein (1962) described how the non-linearity of Einstein's field equations required that in the presence of a static electric or magnetic field, electromagnetic (EM) wave propagation generated an associated coupled GW. In addition, if a static magnetic (or static electric) field is superimposed upon a GW propagating perpendicular to the field direction, the two will interact to generate an EM wave in the same direction as the GW (the inverse Gertsenshtein effect). The amplitude of the generated wave is, however, so small that practical exploitation of this effect is fraught with apparently insuperable difficulties.

The synchro-resonance solution of Einstein's field equations exploited in the Li-Baker detector is different from the Gertsenshtein (1962) effect and is shown schematically in Fig. 2. The proposed Li-Baker detector is sensitive to HFGW directed along the *z*-axis, and when there is a magnetic field along the *y*-axis photon flux will be generated via synchro-resonance along the *x*-axis in either direction. Thus the detector has four major components: a Gaussian microwave beam (GB) directed along the *z*-axis at the frequency of intended GW detection; a static magnetic field **B** directed along the *y*-axis; two fractal membranes used as focusing elements, and high-sensitivity microwave receivers along the *x*-axis. A resonant cavity can be used to enhance the amplitude of the resultant effect. The resultant conversion efficiency is much greater than from the inverse Gertsenshtein effect as exploited in previously proposed HFGW detectors.

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CALCULATING THE STANDARD QUANTUM LIMIT (SQL)

A method for calculating the Standard Quantum Limit (SQL) is introduced in this section. The calculation of coherent versus stochastic SQL is compared and contrasted. Important terms of the SQL calculation are described, including the impact of contained energy levels within the detector on SQL, and the sources of Quality Factor and its effect on SQL.

Coherent versus Stochastic SQL

The question under consideration in this paper is whether or not the Li-Baker detector is quantum-limited when detecting relic HFGW. In other words, does the standard quantum limit (SQL) interfere with the sensitivity of the Li-Baker detector design? The answer will be negative if the SQL is less than 10^{-32} m/m. Grishchuk (1977, 2007) has calculated the SQL for GW detectors in general, which for a coherent GW is:

$$h_{\rm det} = (1/Q)(\hbar\omega/\mathcal{E})^{1/2} \tag{3}$$

and for a stochastic GW is:

$$h_{\rm det} = (1/Q)^{1/2}(\hbar\omega/\mathcal{E})^{1/2}, \tag{4}$$

where $h_{\rm det}$ is the metric (strain) detection limit in m/m, ω is the frequency of sensed gravitational waves (typically around 10GHz in the Li-Baker detector), $\mathcal{E}$ is the effective energy contained within the detector cavity summed over the detection averaging time, and Q is the quality factor or selectivity of the signal over noise. The SQL depends on the values of these parameters. For the remainder of this paper, we will consider the SQL of only the stochastic signal detection case. In the following subsections the best possible value of the SQL using current technology will be estimated to determine the fundamental limitations of the Li-Baker detector as now envisioned.

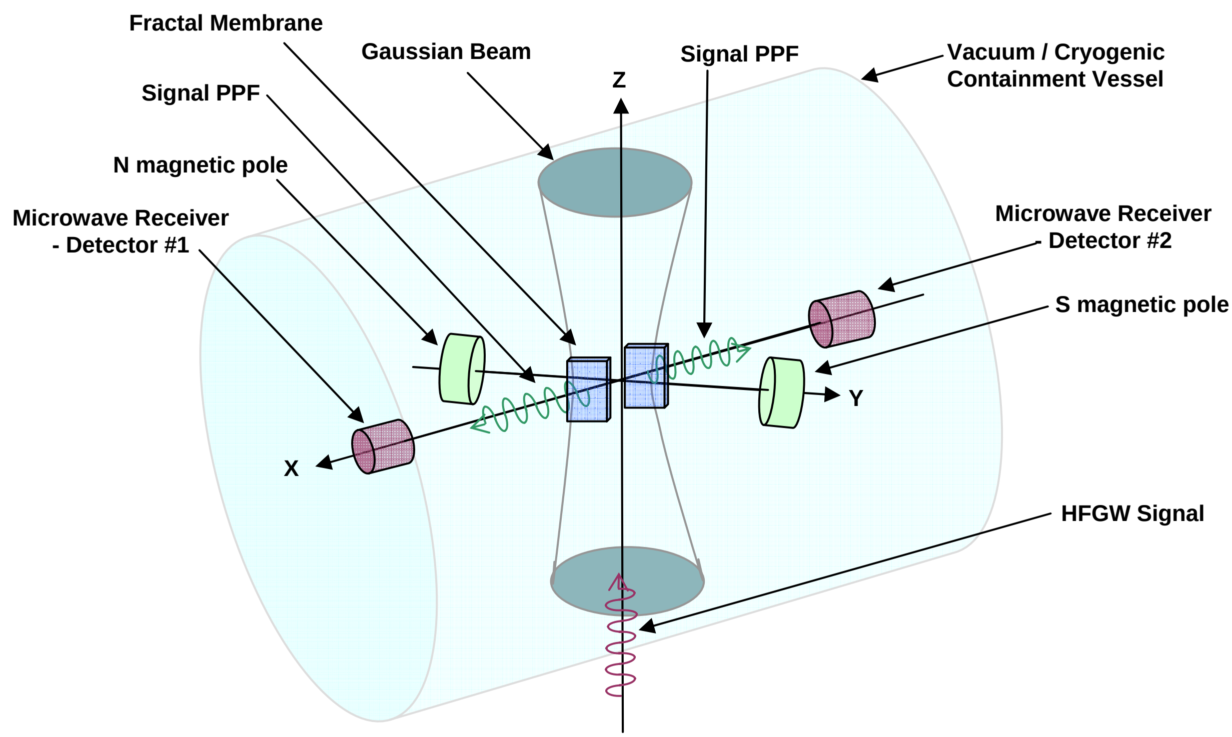

FIGURE 2. The configuration of the Li-Baker Energy Resonance Detection System.

Impact of Contained Energy Levels on SQL

Let us first attempt to estimate a realistic best case for the energy contained within the detection process, $\mathcal{E}$. Typically it is expected that for a refrigerated microwave resonant cavity the best possible electrical quality factor will be around $2\pi\times10^5$. Assuming a "best efforts" value of 1000W for the power of the Gaussian beam in a laboratory installation, the effective total RF energy stored in the microwave resonant cavity of the Li-Baker detector, summed over the system averaging time, is estimated to be given by (Grishchuk, 2007):

$$\mathcal{E}_{\mathrm{RF}} = (10^3\mathrm{W}) \times (1000\mathrm{s}) \times (2\pi\times10^5/2\pi) = 10^{11}\mathrm{J} \tag{5}$$

over a typical 1000s averaging time. Both the Li-Baker detector and a detector using the Gertsenshtein effect use a large static magnetic field B. For the present suggested outline design for the Li-Baker detector, the nominal value of B = 3T, so that the magnetic energy density is given by

$$\mathcal{E}_{\mathrm{B}} = (\tfrac{1}{2})B^2/(\mu_r\mu_o) = 3.6\times10^6\mathrm{J\ m}^{-3}. \tag{6}$$

The interaction volume in a practical laboratory-based detector is likely to be a maximum of around $1\mathrm{m}^3$. So, the effective total stored energy from the Gaussian beam is much greater than the stored magnetic field energy, and it follows that $E \approx E_{RF} = 10^{11}$J to a reasonable approximation.

Sources of Quality Factor and Effect on SQL

To calculate the SQL, h_{det}, we also need the value of the detector quality factor Q (not the same as the cavity quality factor). Anything that concentrates or enhances the signal preferentially over noise, in any measurement dimension, can be considered a contributor to the quality factor Q. The quality factor can therefore be understood as the "signal selectivity" in each dimension, so that

$$Q_{\mathrm{tot}} = (Q_{\mathrm{spatial}})(Q_{\mathrm{t}}) = Q_r Q_{\mathrm{solid\ angle}} Q_t. \tag{7}$$

The temporal quality factor in the Li-Baker detector arises from averaging the signal over time, so that at 10GHz, $Q_t = \Omega.t_{\mathrm{int}} = 10\times10^9\mathrm{Hz} \times 1000\mathrm{s} = 10^{13}$.

There is a contribution to Q arising from the fractal membranes that focus and concentrate the signal photon energy – but not the background photons – along the radial dimension. The radial selectivity arising from the use of fractal membranes is calculated by Li *et al.* (2008). Their table III gives $Q_r = \mathrm{SNR}_{(r=37\mathrm{cm})}/\mathrm{SNR}_{(r=3.5\mathrm{cm})} = 3.4\times10^{21}$.

Also, there is an effective Q contribution arising from the synchro-resonance solution to the Einstein field equations that limit the PPF signal to a radiation pattern in certain directions, whereas noise is distributed uniformly. By utilizing directional antennas, the Li-Baker detector can capitalize upon this as a contribution to Q in angular space. This is calculated in detail, octant by octant, by Li *et al.* (2008). Page 24 of Li *et al.* summarizes this in terms of angular concentration onto the detector. A non-directional antenna corresponds roughly to solid angle 2π steradians (one hemisphere), so that the effective antenna gain is estimated as $(Q_{\mathrm{solid\ angle}}) = 2\pi\ \mathrm{sr}/10^{-4}\mathrm{sr} = 6.3\times10^4$. Therefore, the predicted maximum quality factor will be $Q_{\mathrm{total}} = Q_r Q_{\mathrm{solid\ angle}} Q_t = 2.1\times10^{39}$. This finally gives the Standard Quantum Limit (SQL) for stochastic GW detection at 10GHz:

$$h_{\mathrm{det}} = (1/Q)^{1/2}(\hbar\omega/\mathcal{E})^{1/2} = 1.8\times10^{-37}\mathrm{m/m}. \tag{8}$$

STANDARD QUANTUM LIMIT COMPARISONS

The results of the Standard Quantum Limit calculation for the Li-Baker detector are compared with the predicted sensitivity of this detection system in this section. A comparison of Li-Baker detector with Gertsenshtein Effect detector is also made.

Comparison of SQL with Predicted Sensitivity

As noted in the previous section, $h_{\mathrm{det}} = 1.8\times10^{-37}$m/m represents the lowest possible GW amplitude detectable by each RF receiver in the Li-Baker HFGW detector, limited by quantum back-action. An additional (1/√2) factor applies if the separate outputs from the two RF receivers are averaged, rather than used independently for false alarm reduction, resulting in a minimum $h_{\mathrm{det}} = 1.2\times10^{-37}$. Since the predicted best sensitivity of the Li-Baker detector in its currently proposed configuration is $A = 10^{-32}$m/m, these results confirm that the Li-Baker Detector is photon-signal limited, not quantum noise limited; that is, the Standard Quantum Limit is so low that a properly designed Li-Baker detector can have sufficient sensitivity to observe HFRGW of amplitude $A \approx 10^{-32}$m/m.

Comparison of Li-Baker detector with Gertsenshtein Effect detector

As described in Stephenson (2005) and Gertsenshtein (1962) a magnetic field alone could act as a GW detector in a vacuum via synchro-resonance. However, unlike the Li-Baker solution of a synchro-resonance, since the Gertsenshtein solution uses only a magnetic field to couple to the GW, there is much less energy contained within the detection process, which will therefore lead to a larger SQL. The Q factor is also much lower for Gertsenshtein Effect Detector:

Because the Gertsenshtein detector is a passive device, the only source of Q is the integration of signal over time,

$$Q_t = \Omega \times t_{int} = (10(10)^{+9}\ \mathrm{Hz}).(10^3\mathrm{sec}) = (10)^{+13} \qquad (9)$$

for a 10 GHz HFGW signal. Therefore $Q_{tot} = (10)^{+13}$. The SQL calculation for Gertsenshtein Effect Detector is therefore:

$$h_{\mathrm{det}} = (1/Q)^{1/2}(\hbar\omega/\varepsilon_B)^{1/2} = (4.5(10)^{-7}).[(6.6(10)^{-34})(10(10)^{+9})/3(10)^{+6}]^{1/2} \qquad (10)$$

Which gives $h^{\mathrm{det}} = 6.7(10)^{-22}$ m/m for the Gertsenshtein detector for a stochastic signal at 10GHz. This result implies that the SQL will adversely limit Gertsenshtein detector performance if it configured with the magnetic energies specified here.

But in contrast, the SQL does not appear to affect the operation of the Li-Baker detector design at the currently specified sensitivities. Comparing the Li-Baker Detector SQL to Gertsenshtein Detector SQL, we find that $SQL_{LB} = h_{\mathrm{det}} = 1.8(10)^{-37}$, versus $SQL_G = h_{\mathrm{det}} = 6.7(10)^{-22}$. Thus there is a 16 order of magnitude difference calculated between the two detector types. Why is there such a significant difference between the two designs? Improvements in the Li-Baker detector over the Gertsenshtein detector design may be accounted for by: first, improvements from fractal membrane use, accounting for a factor of $\sim10^{-10}$ in sensitivity, second, improvements from geometric “antenna gain,” accounting for a factor of $\sim10^{-2}$, and finally a larger detector energy content, providing another $\sim10^{-4}$. So we see that while energy content of the detection system is important, most of the improvements are derived from quality factor differences.

CONCLUSION

The results of the Standard Quantum Limit (SQL) calculation in this paper imply that the Li-Baker Detector is Photon signal limited, not quantum noise limited, since SQL = $h_{\mathrm{det}} = 1.8(10)^{-37}$, versus a predicted sensitivity per EPJC 2008 of $h_{\mathrm{det}}=(10)^{-32}$, it is therefore concluded that the Standard Quantum Limit is sufficiently low to theoretically allow the operation of the Li-Baker detector in the sensitivity range of relic HFGW, $\sim(10)^{-32}$ m/m. This compares favorably with the Gertsenshtein style passive detector, which was calculated to have a higher SQL of $h_{\mathrm{det}} = 6.7(10)^{-22}$ for the case of stochastic signal detection.

NOMENCLATURE

B = magnetic field strength (T)
ε_B = magnetic field energy (J)
h_{det} = lower detection limit of metric strain (m/m)
$\hbar$ = Plank's constant $6.634x(10)^{-34}$ (J/s) / 2π
Q = quality factor (unitless)
Q_r = quality factor, radial (unitless)
$Q_{solid\ angle}$ = quality factor, angular (unitless)
Q_t = quality factor, temporal (unitless)
Q_{tot} = quality factor, total (unitless)
$SNR_{(r=3.5cm)}$ = signal to noise ratio, at radius r=3.5cm
$SNR_{(r=37cm)}$ = signal to noise ratio, at radius r=37cm
SQL = standard quantum limit (unitless)
SQL_G = standard quantum limit, for Gertsenshtein detector (unitless)
SQL_{LB} = standard quantum limit, for Li-Baker detector (unitless)
t_{int} = integration time (s)
ω = frequency (radians/sec)
Ω = frequency (Hz)

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